Roll No. [ ] Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(ME) (2012 Onwards) (Sem.–5)

MATHEMATICS-III

Subject Code : BTAM-500

M.Code : 70601

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.

SECTION-A

1. Write briefly :

a) Expand $f(z)=\dfrac{z}{(z+1)(z+2)}$ about $z = -2$.

b) Evaluate $\int_C \dfrac{\sin z}{z\cos z}dz$ along the circle C : $|z| = 2$.

c) Find the bilinear transformation that map the points $z = 1, -i, -1$ into the points $w = i, 0, -i$.

d) Find $L(t^2 \sin 3t)$.

e) Form a partial differential equation from $z = f(x + 4t) + g(x - 4t)$.

f) Find the solution of homogeneous partial differential equation $2r - 5s + 2t = 0$.

g) Write Dirichlet's conditions for the expansion of $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.

h) Show that $P_n(-x) = (-1)^n P_n(x)$.

i) State Cauchy's Residue theorem.

j) Find the coefficient a_0 in the Fourier series of $f(x) = |x|, -\pi < x < \pi$.

SECTION-B

2. Prove that $\int J_3(x)\,dx = -J_2(x) - \frac{2}{x} J_2(x)$.

3. Expand $f(x) = x \sin x,\ -\pi < x < \pi$ as a Fourier series.

4. State convolution theorem and hence evaluate $L^{-1}\left[\frac{s^2}{(s^2+4)(s^2+9)}\right]$.

5. If $f(z) = u + iv$ is an analytic function, then find $f(z)$ if $u+v=\frac{x}{x^2+y^2}$.

6. Solve the following partial differential equation by method of separation of variables :

$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u, \text{ Given that } u = 3e^{-y} - e^{-5y} \text{ when } x = 0$$

SECTION-C

7. Use the concept of residues to evaluate $\int_0^{\pi} \frac{d\theta}{(a+b\cos\theta)}$, where $a > |b|$.

8. A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \lambda x(1 - x)$, where λ is constant and then released. Find the displacement of any point x of the string at any time $t > 0$.

9. Solve in series the equation :

$$(1 + x^2)\, y'' + xy' - y = 0$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.